AI Olympiad, February 2026

1. Let $X$ be a finite set of some finite size $k$. Let $A$ be a learner that on a sample $S = ((x_1, y_1), \ldots, (x_m, y_m)) \in (X \times \{0,1\})^m$ outputs a function $A(S)$ such that:

   If $x = x_i$ for some $i \in \{1, \ldots, m\}$ then $A(S)(x) = y_i$,

   If $x \notin \{x_1, \ldots, x_m\}$, then $A(S)(x)$ is picked randomly with probability $1/2$ for each label in $\{0,1\}$.

   Given a function $f : X \to \{0,1\}$, define a probability distribution $(U, f)$ over $X \times \{0,1\}$ by assigning probability $1/k$ to any pair $(x, f(x))$ and probability 0 to any $(x, y) \notin \{(x, f(x)) : x \in X\}$.

   Show that for every $f : X \to \{0,1\}$ and $S = ((x_1, f(x_1)), \ldots, (x_m, f(x_m)))$,

$$L_{(U,f)}(A(S)) - L_S(A(S)) \geq \frac{(k-m)}{2k}.$$

   [7 Marks]

[Total 7 Marks]