AI Olympiad, February 2026

1. a) Express the distance of the image of a point $\mathbf{x}$ from the centre of mass of a set of examples

$$S = \{\mathbf{x}_1, \ldots, \mathbf{x}_m\}$$

in a feature space defined by kernel $\kappa(\mathbf{x}, \mathbf{z}) = \langle \phi(\mathbf{x}), \phi(\mathbf{z}) \rangle$ in terms of kernel evaluations.

[2 Marks]

b) Using the result of part (1a) give pseudocode for the novelty detection algorithm that labels a test point novel if it lies outside the smallest sphere centred at the centre of mass that contains all of the training data.

[3 Marks]

c) Show that if we remove one point $\mathbf{x}_j$ from S, the centre of mass moves away from $\phi(\mathbf{x}_j)$ on the line from $\phi(\mathbf{x}_j)$ through the centre of mass by $\frac{1}{m-1}$ times the distance between them.

[3 Marks]

d) Hence or otherwise show that the novelty detection method using the same centre as in part (1b) but increasing the radius of the sphere by $\frac{m}{m-2}$ ensures that the leave one out error (an error occurs if the left out point is not inside the sphere) on the training set is at most 1.

[5 Marks]

[Total 13 Marks]